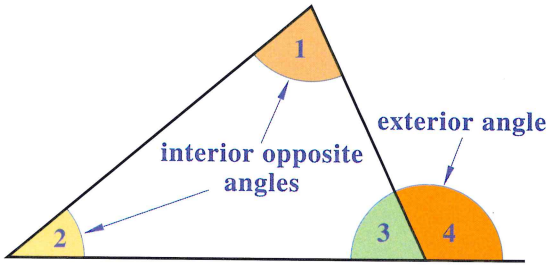
Exterior Angles of a Triangle

An **exterior angle** of a triangle is formed when one side is extended through a vertex. In the figure over, the base of the triangle is extended to form an **exterior angle**.

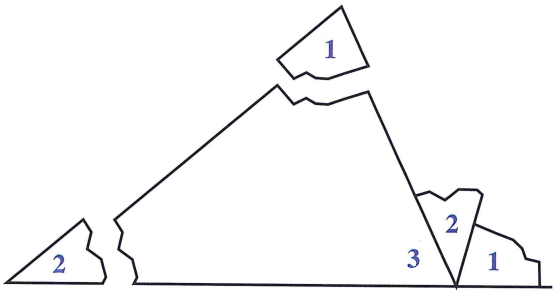


The exterior angle at any vertex in a triangle is equal to the sum of the two interior opposite angles.

In the figure across, $\angle 4 = \angle 1 + \angle 2$.

The following activity is an informal proof of the theorem:

1. Cut out a triangle of any shape and size from a sheet of paper.
2. Cut off the two angles opposite the exterior angle and arrange these at the exterior angle as shown across.
3. Notice that the two interior opposite angles together form the exterior angle.



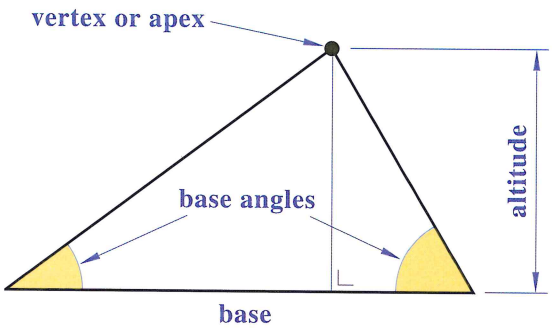
Answer Worksheet 4A

Properties of Triangles

The **vertex** or **apex** is the point of the angle opposite the **base**.

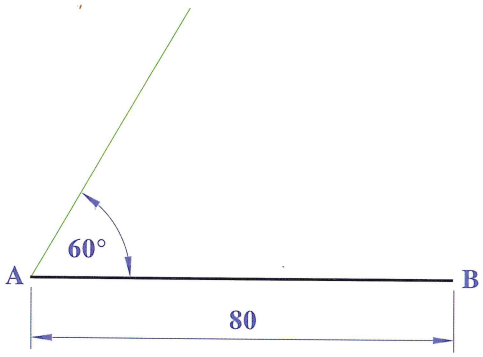
The **altitude** is the perpendicular distance from the apex to the base.

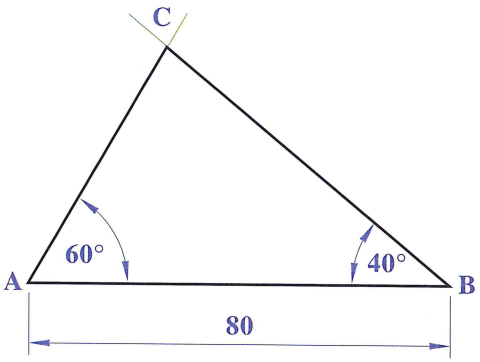
The angles at the base are called the **base angles**.



Example 1

Construct a **triangle** having a base of length 80 mm and **base angles** of 60° and 40°.

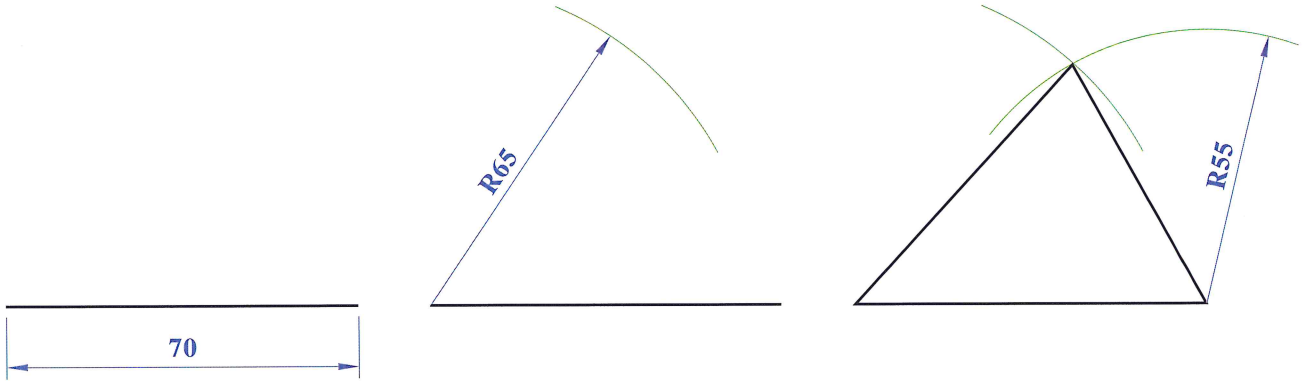




1. Draw the **base** AB 80 mm in length.
2. Using the 60°/30° set square, construct a 60° angle at A.
3. Using a protractor, construct a 40° angle at B, cutting the 60° line at C. Then ABC is the required triangle.

Example 2

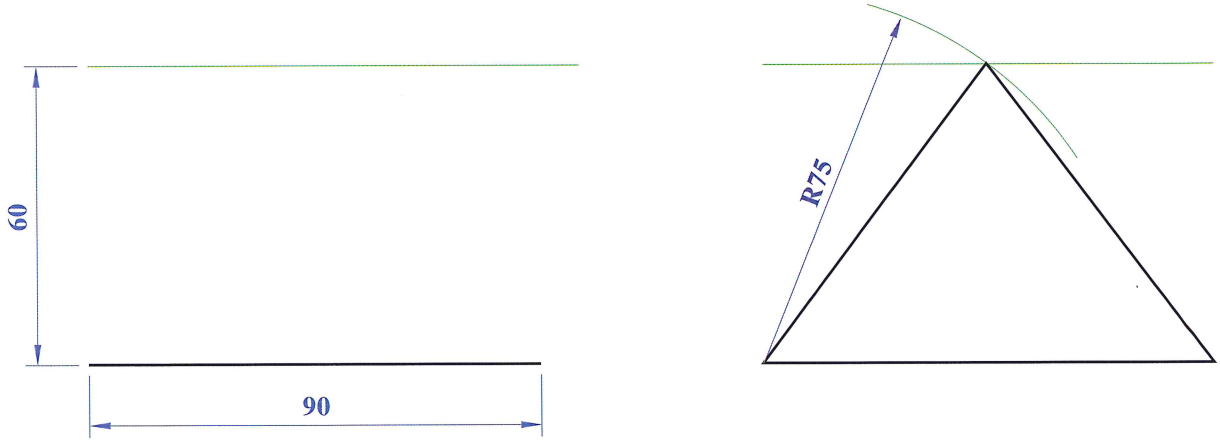
Construct a **triangle** having sides of length 70 mm, 65 mm and 55 mm.



1. Draw the **base** of the triangle of length 70 mm.
2. With the point of the compass at one end of the base, swing an arc of radius 65 mm.
3. Swing an arc of radius 55 mm from the other end of the base to intersect the first arc. Draw the required triangle.

Example 3

Construct a **triangle** having a **base** of length 90 mm, a side of length 75 mm and an **altitude** of 60 mm.



1. Draw the **base** of the triangle, 90 mm in length.
2. Draw a line parallel to the base and a distance of 60 mm from it. This is the **altitude**.
3. Swing an arc of radius 75 mm from the other end of the base to intersect the altitude. Draw the required triangle.

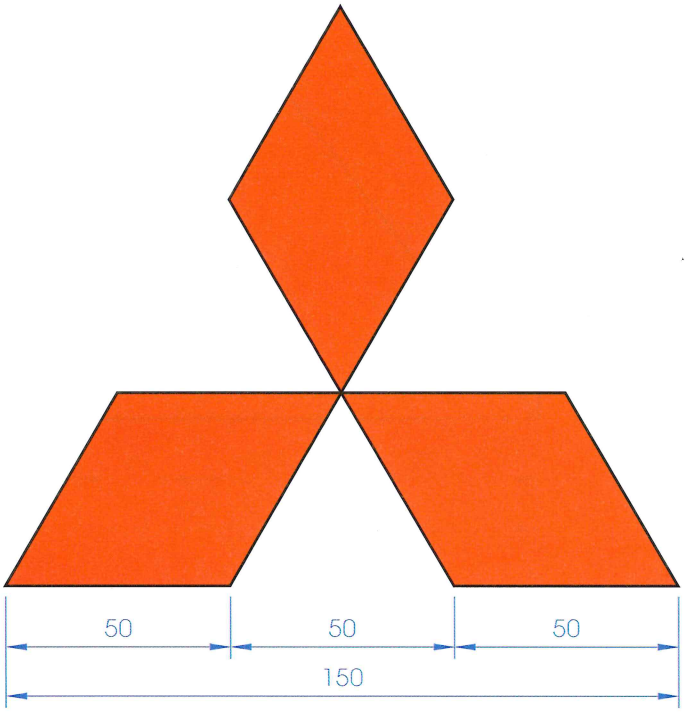
Always use this parallel line method when the altitude is given.

Answer Worksheet 4B

Exercises

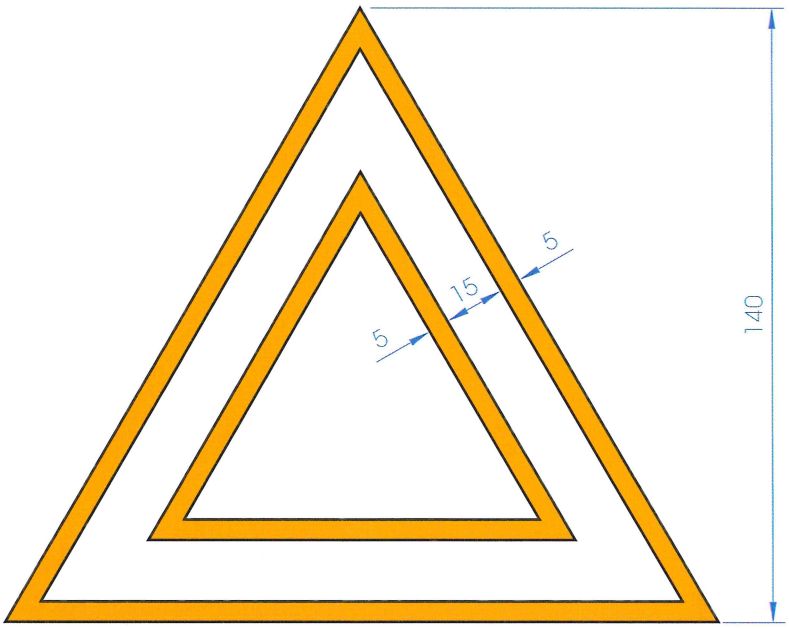
1. The **Mitsubishi** logo shown below has become one of the most widely used logos in the world. The basic shape is an **equilateral triangle** of side 150 mm.

Draw the logo. Using a coloured pencil, apply shading to your drawing.



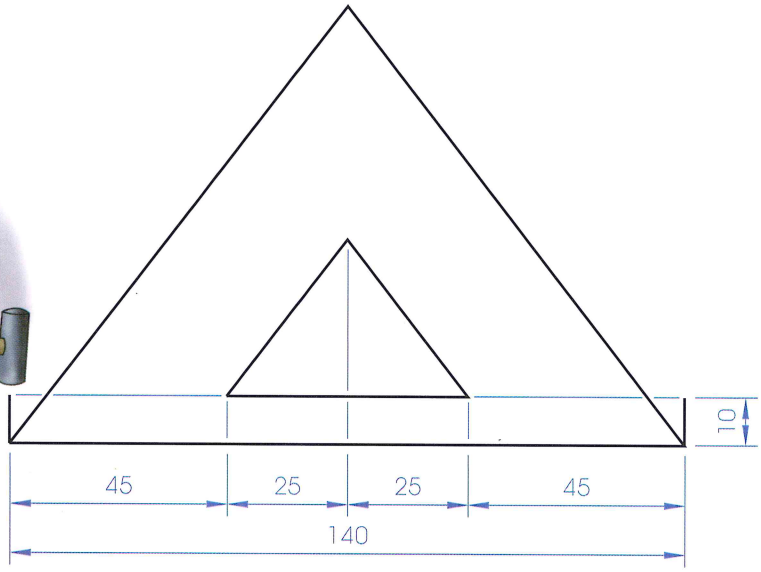
2. The drawing shown in the figure below is the symbol for a **hazard light** switch in a car. It is composed of a series of **equilateral triangles**. The **altitude** of the outer triangle is 140 mm.

Reproduce the drawing full-size. Apply shading to your drawing with a coloured pencil.



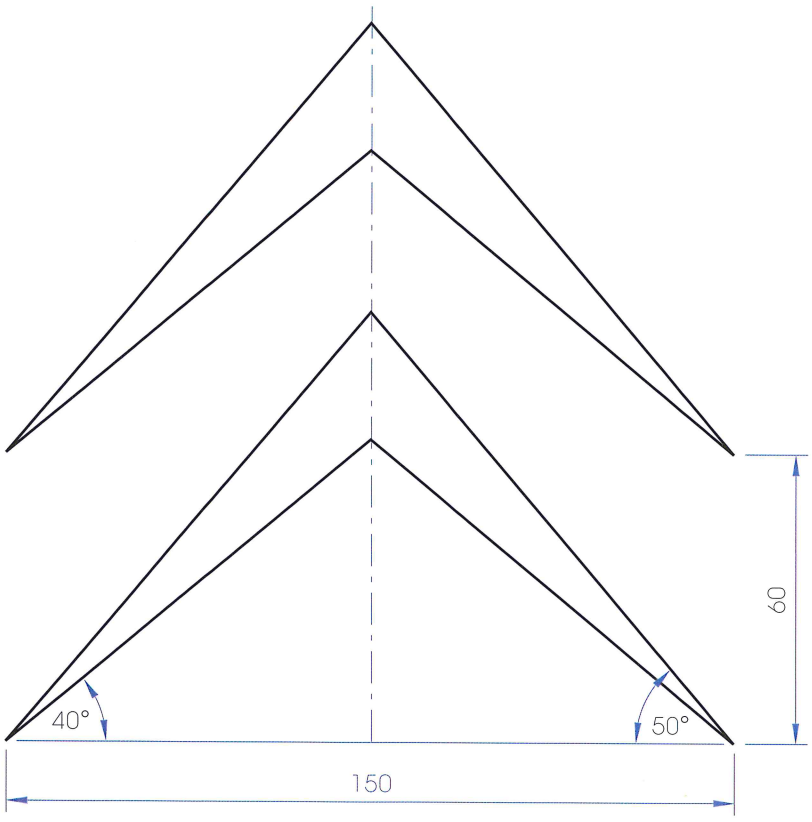
3. A sign for a **camp site** is shown in the figure below. It contains two **isosceles triangles** which are **similar** (see page 33). The base of the larger triangle is 140 mm long and the two equal sides are each of length 115 mm.

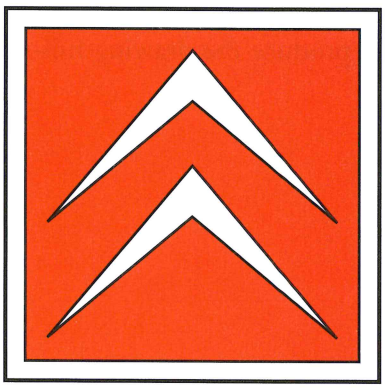
Make a full-size drawing of the sign showing all construction lines.



4. A drawing of the symbol for the **Citroen** car company is shown below. Notice that the symbol contains two sets of **isosceles triangles**.

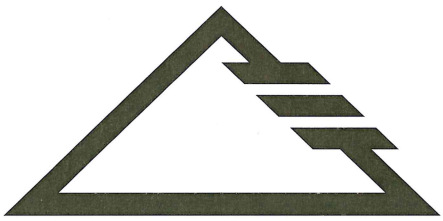
Using the measurements given, make a full-size drawing of the symbol.



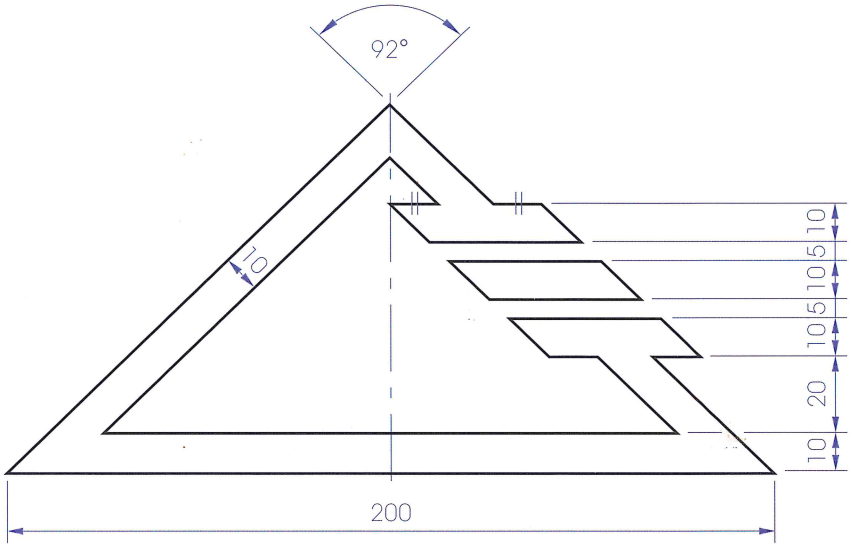


5. The drawing of the **Irish Insurance Federation** symbol shown across is based on an **isosceles triangle**.

Make a copy of this drawing.

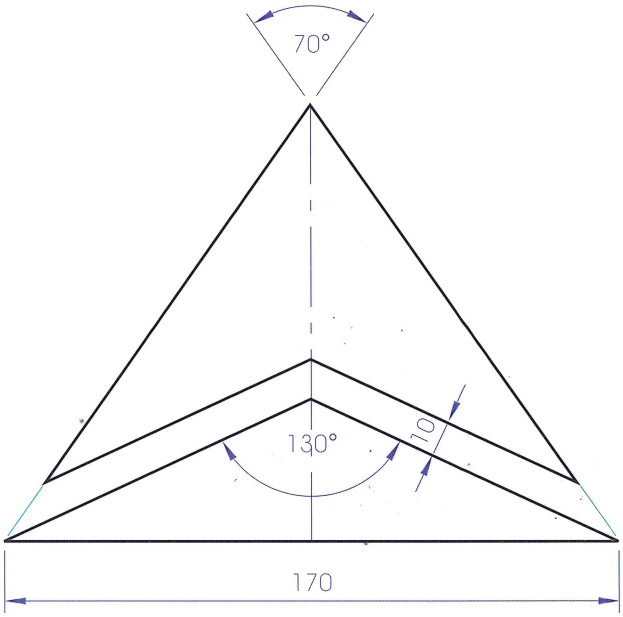


Irish Insurance Federation

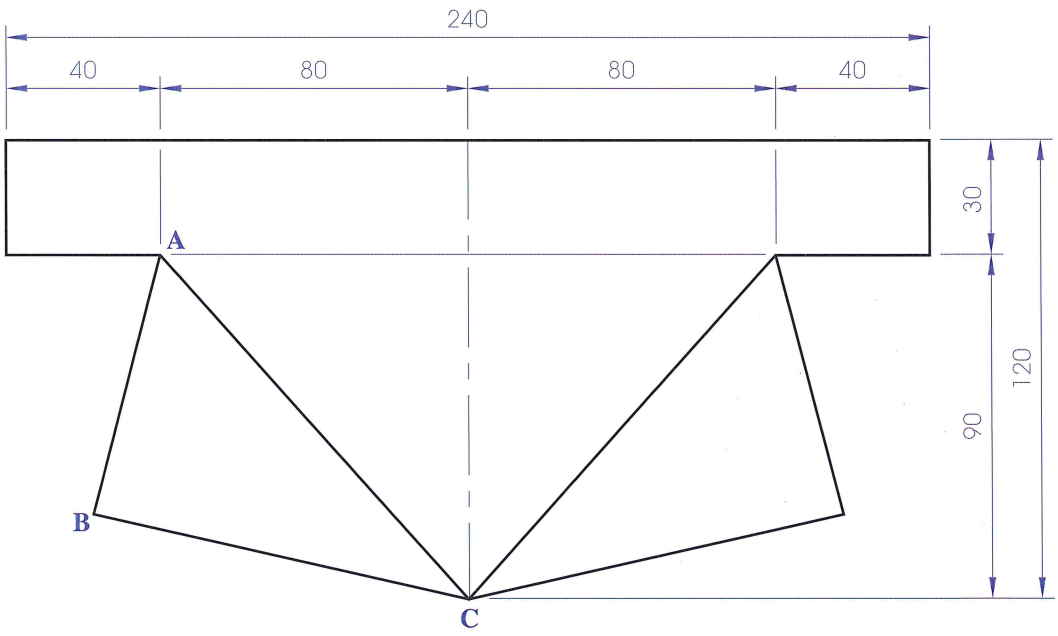


6. A drawing of the **Delta Airlines** emblem is shown across. You will notice that the emblem contains two interlocking **isosceles triangles**.

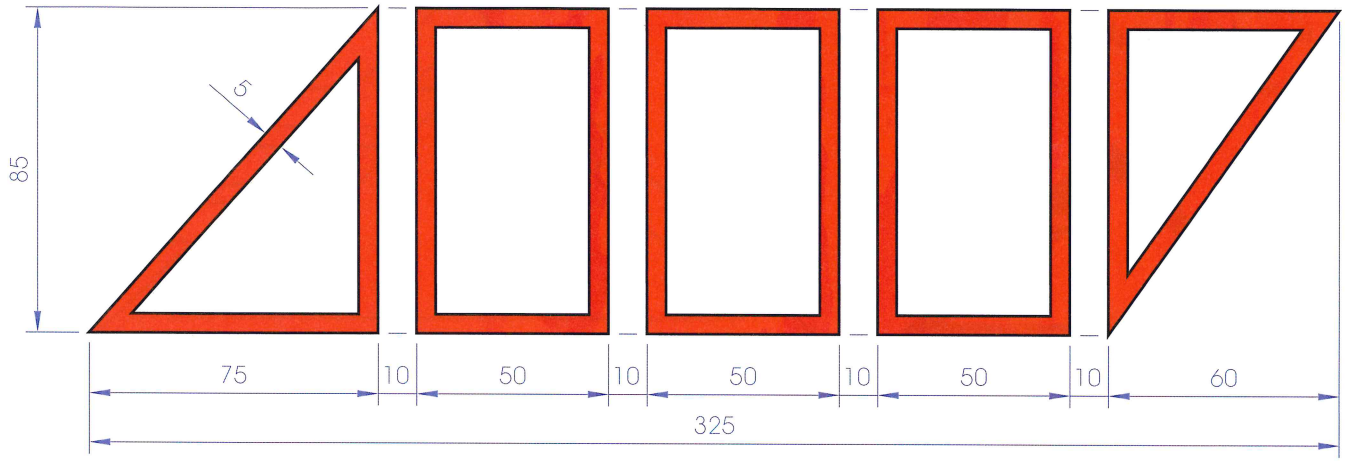
Draw the emblem.



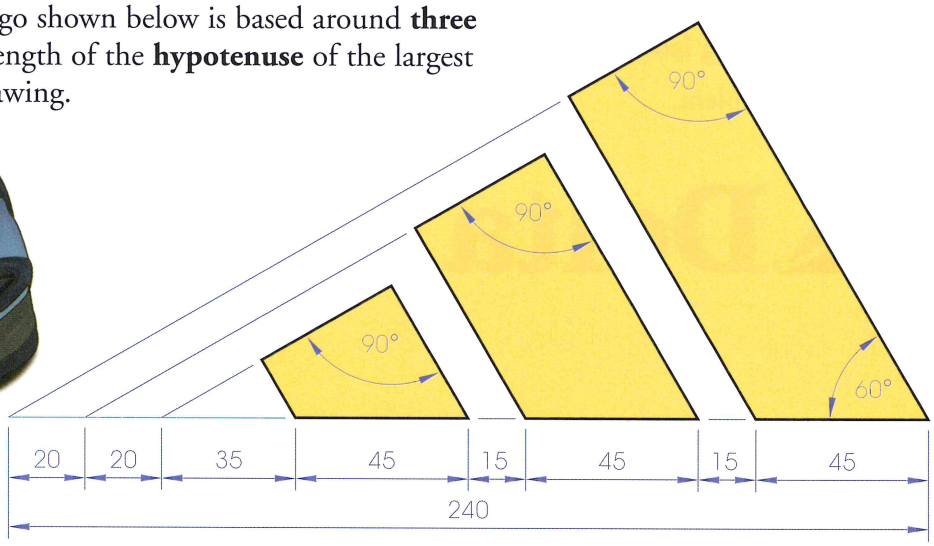
7. Make a full-size drawing of the **Blacktie** logo shown below. The sides AB and BC of the **scalene** triangle ABC are of lengths 70 mm and 100 mm, respectively.



8. A drawing of the **Autoglass** sign is shown below. It is made up of **right-angled triangles** and **rectangles**. All margins are 5 mm. Copy the drawing.

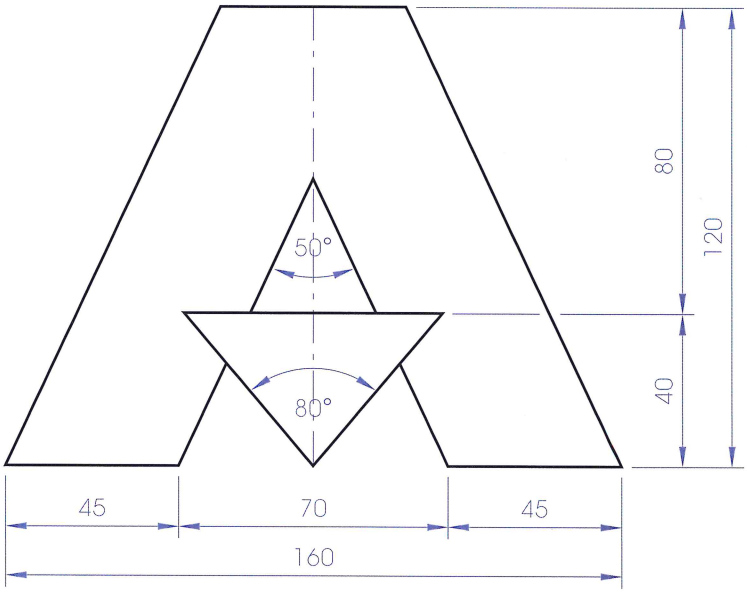


9. The drawing of the **Adidas** logo shown below is based around **three right-angled triangles**. The length of the **hypotenuse** of the largest triangle is given. Copy the drawing.



10. The symbol for **A Plant** shown below contains two **isosceles triangles**.

Reproduce the drawing from the measurements given.

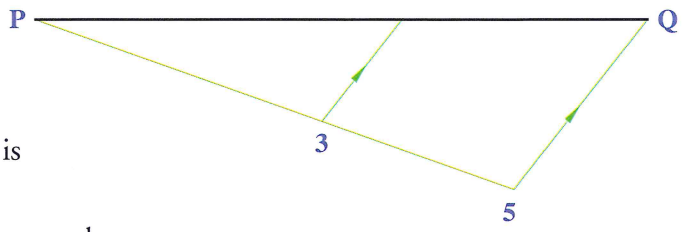


Ratios in Geometry

Ratios are used to compare quantities. For example, if the lengths of two line segments are 10 mm and 30 mm respectively, the ratio of the lengths is 1:3. We can be asked to divide a line in a given ratio.

Dividing a Line in a Given Ratio

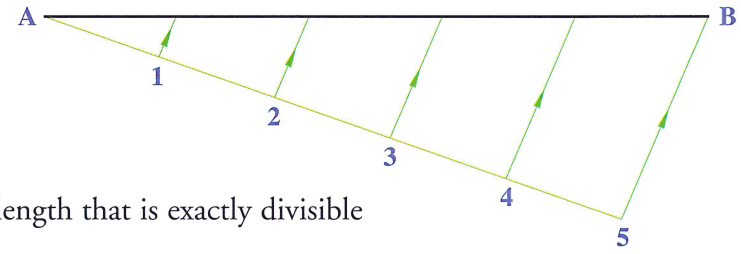
Example
Divide the given line PQ in the ratio 3:2.



1. Draw a line at an angle to PQ and of a length that is exactly divisible by five (3 + 2) – say 50 mm.
2. Mark off the line in the ratio 30:20 which is the same as the ratio 3:2. Join the end of the line to Q.
3. Draw a line parallel to this line through point 3, cutting PQ in the ratio 3:2.

Dividing a Line into a Number of Equal Parts

Example
Divide the given line AB into five equal parts.



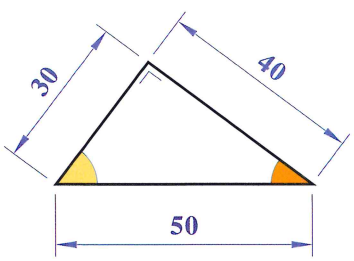
1. Draw a line at an acute angle to AB and of a length that is exactly divisible by five (say 100 mm).
2. Mark off the line into five equal parts (100 ÷ 5 = 20 mm). Join the end of the line to B.
3. Draw lines parallel to this line through points 4, 3, 2 and 1, dividing AB into five equal parts.

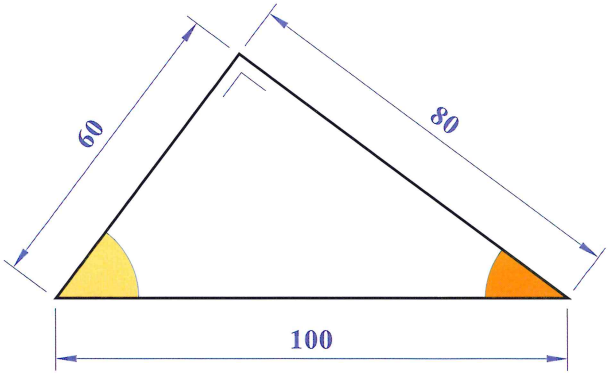
Answer Worksheet 4C

Similar Triangles

Similar triangles are triangles that are the same shape, but not the same size.

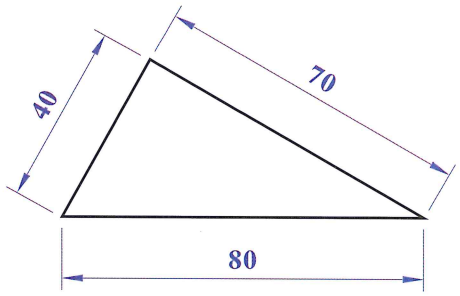
When two triangles are similar, the angles of one are the same size as the angles of the other. In similar triangles, the sides of each triangle are in the same ratio. In the figure below, the sides of the similar triangles are in the same ratio 3:4:5.





Perimeter of a Triangle

The perimeter of a triangle is the sum of the lengths of the three sides.

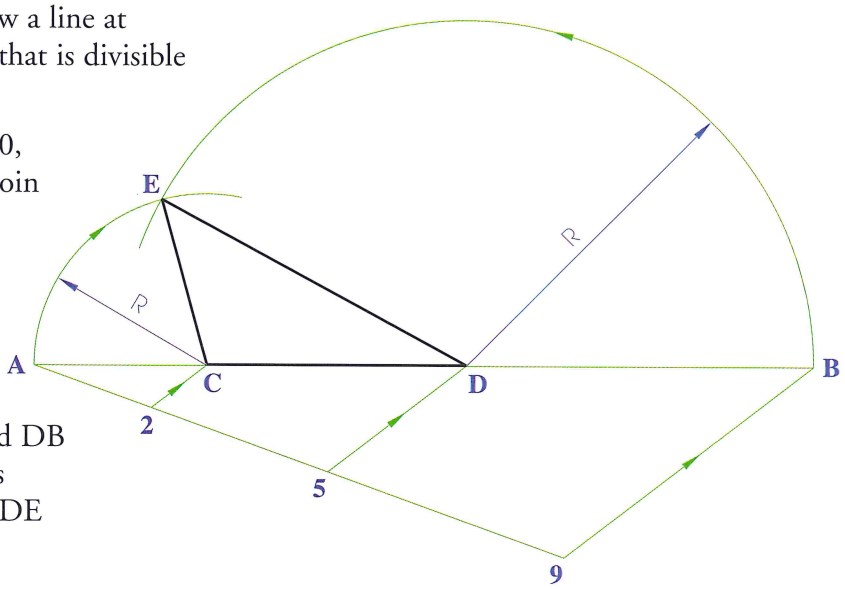


Thus, the **perimeter** of the triangle over
= 80 + 70 + 40
= 190 mm

Example 1

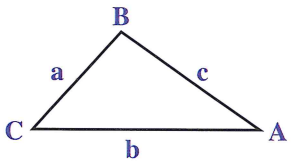
Construct a triangle having a perimeter of 125 mm and having sides in the ratio 2:3:4.

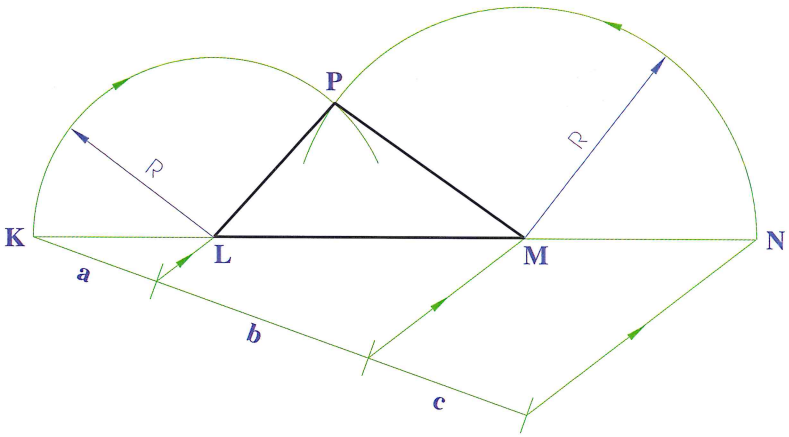
1. Draw the line AB 125 mm long. Draw a line at an acute angle to AB and of a length that is divisible by 9 (2 + 3 + 4) – say 90 mm.
2. Mark off the line in the ratio 20:30:40, which is the same as the ratio 2:3:4. Join the end of the line to B.
3. Draw lines parallel to this line through the points 2 and 5, cutting AB at the points C and D. Draw the base CD of the triangle.
4. Using C and D as centres and CA and DB as radii respectively, draw the two arcs intersecting at E. Join CE and DE. CDE is the required triangle.



Example 2

Construct a triangle similar to ABC having a perimeter of 115 mm.





1. Draw the line KN 115 mm long and divide it in the ratio a:b:c.
2. Draw the base LM of the triangle.
3. Using L and M as centres and LK and MN as radii respectively, draw the two arcs intersecting at P. Join LP and MP. LMP is the required triangle.

Answer Worksheet 4D